California's Common Core *Mathematics Framework*

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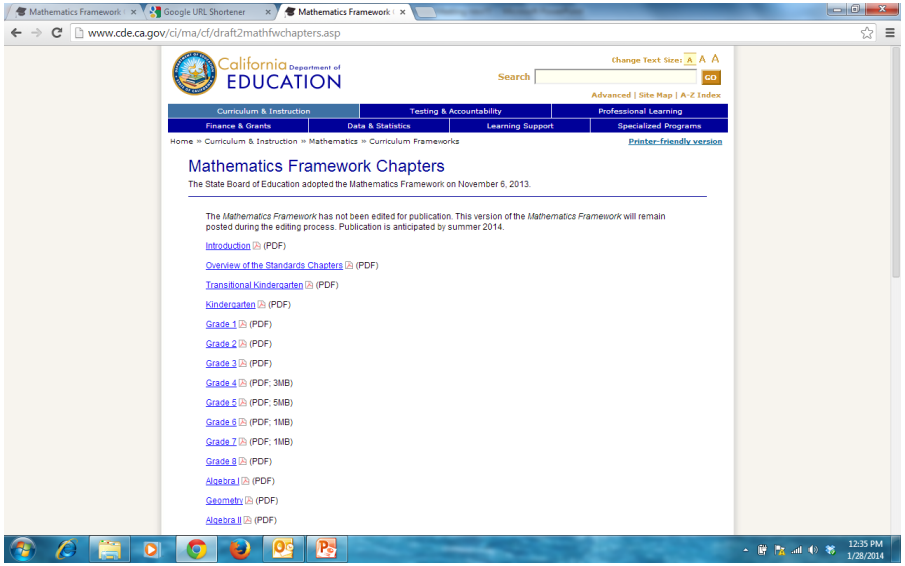
Resources:

Mathematics Framework Chapters

Link: http://goo.gl/wlg6l1

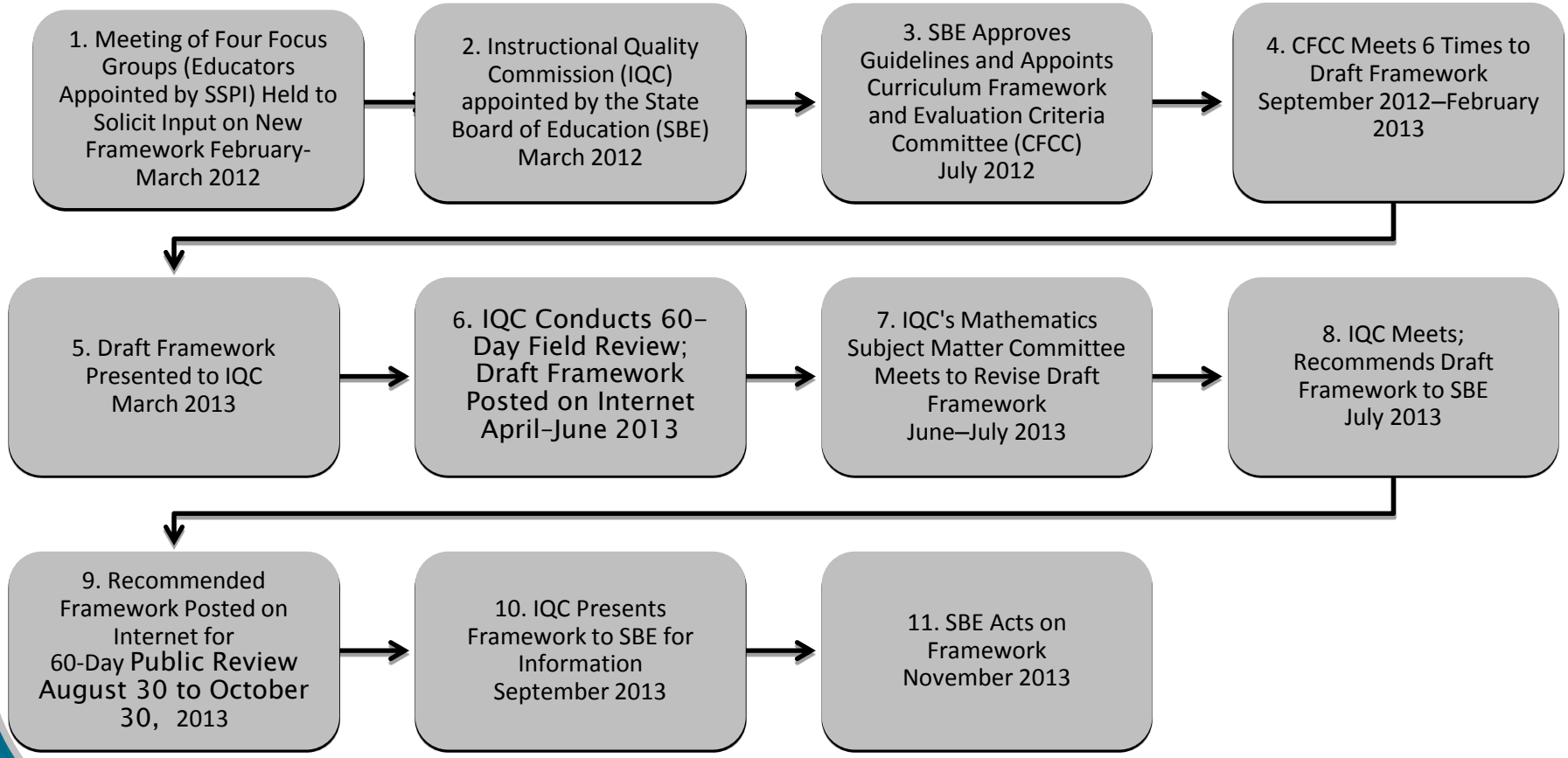


CALIFORNIA DEPARTMENT OF EDUCATION

Mathematics Framework
Development Process

This chart shows the major steps of the curriculum framework development process.

All meetings are open to the public.



Goals for the *Mathematics Framework*

- Guide the field in implementing the CA CCSSM
- Emphasize coherence across and within grade levels
- Integrate the Mathematical Practice and Content Standards
- Provide guidance on the higher mathematics course progression

What is in the *Mathematics Framework*?

- Introduction
- Overview of Standards Chapters
- Grade-level chapters, TK–8
- Higher mathematics chapters by course:
  - Traditional pathway
  - Integrated pathway
  - Pre-calculus, Statistics and Probability
  - Advanced Placement Probability and Statistics
  - Calculus
  - Mathematical Modeling

What is in the *Mathematics Framework*?

- Universal Access
- Instructional Strategies
- Supporting High-Quality Common Core Mathematics Instruction
- Technology in the Teaching of Mathematics
- Assessment
- Instructional Materials to Support the CA CCSSM (including the evaluation criteria for the mathematics adoption)

What is in the Appendix?

1. Course Placement and Sequences
2. Financial Literacy and Mathematics Education
3. Possible Adaptations for Students with Learning Difficulties in Mathematics
4. Mathematical Modeling
5. Higher Mathematics Pathways Standards Chart
6. Method Used for Solving Single-digit Addition and Subtraction Problems

What Guided the Revision of the *Mathematics Framework*?

- National documents and research from the Common Core State Standards Initiative
- Achieve the Core and the Progressions Documents
- The Standards for Mathematical Practice
- State Board of Education Guidelines

SBE Guidelines for the Revision of the *Mathematics Framework*

- Based on input from the focus group meetings, written comments received, and statutory requirements
- Reviewed and recommended by the IQC and approved by the SBE
- The Mathematics CFCC members develop a framework based on the guidelines

Who contributed to the *Mathematics Framework*?

- Focus Group members—all educators in California K–12 public schools, four regional meetings
- MCFCC members—one-half teachers, including teachers with experience teaching English learners and students with disabilities, other educators, and two content experts with Ph.Ds. in mathematics
- IQC—teachers, curriculum leaders, and administrators
- Staff of the Curriculum Frameworks and Instructional Resources Division and mathematics expert Dr. Christopher Yakes

Who contributed to the *Mathematics Framework*?

- The field—provided comments on the draft framework during two 60-day review periods
- County Offices of Education—held discussion forums on the 1st draft of the *Mathematics Framework*
- Common Core State Standards for Mathematics author and expert Jason Zimba
- WestEd's California Comprehensive Center, Neal Finkelstein and Dona Meinders
- Staff of the California Department of Education's Language Policy and Leadership Office, STEM Office, and Assessment Transition Office

The Common Core State Standards Requires Three Shifts in Mathematics

1. **FOCUS:** Focus strongly where the standards focus.
2. **COHERENCE:** Think across grades and link to major topics.
3. **RIGOR:** In major topics, pursue conceptual understanding, procedural skill and fluency, and application.

SHIFT #1: Focus Strongly Where the Standards Focus

- Narrow the scope of the content and deepen how much time and energy is spent in the classroom

- Focus deeply on what is emphasized in the standards, so that ALL students gain strong foundations

IMPLICATIONS:

✓Move away from "mile wide, inch deep" curricula as identified in TIMSS

✓Teach less, learn more!

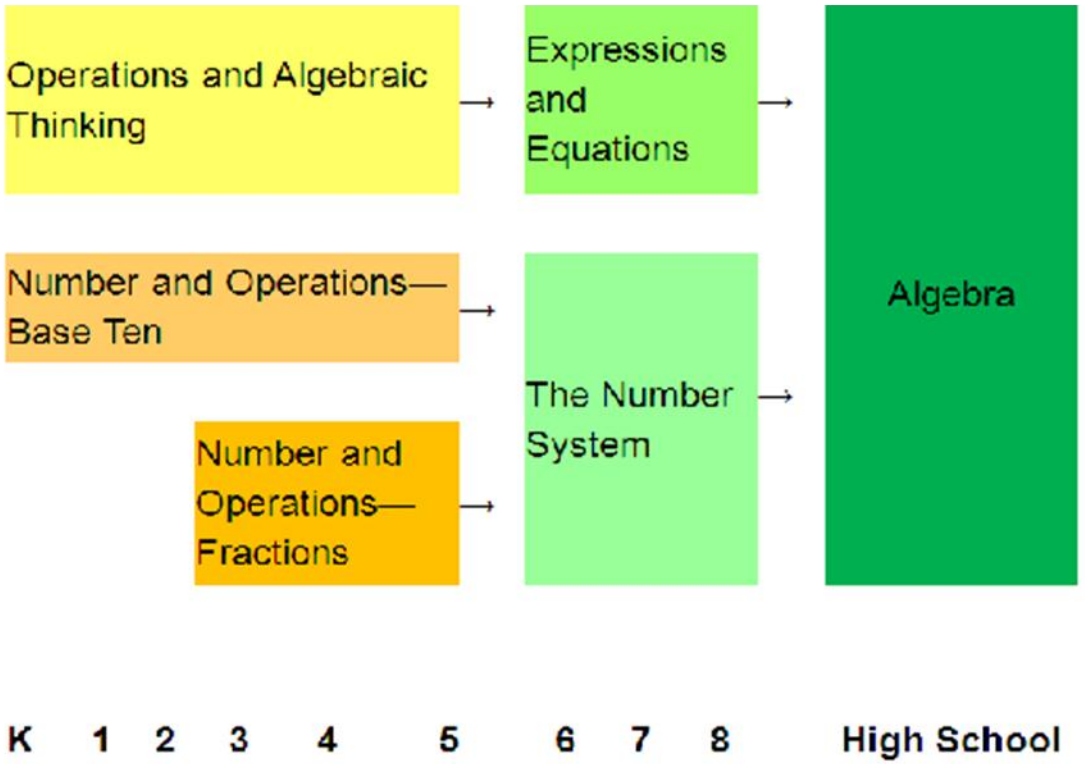
Move from the Traditional U.S. Approach

Covering All Strands Equally in K-8

- Number & Operations
- Measurement & Geometry
- Algebra & Functions
- Statistics & Probability

TO

Focusing on Key Concepts to get students ready for Algebra



Group Discussion

Shift #1: Focus Strongly where the Standards Focus.

In your groups, discuss ways to respond to the question, "Why focus? Why limit students to just a few concepts?

SHIFT #2: COHERENCE: Think Across Grades, and Link to Major Topics Within Grades

- Carefully connect the learning within and across grades so that students can build new understandings on foundations built in previous years.

- Build on solid conceptual understanding of core content. Each standard is NOT a new event but an extension of previous learning.

IMPLICATIONS:

✓**Do not repeat the previous year's content but build on it where applicable**

For example, students might have learned about the decimal 0.25 show them that is equivalent to 1/4 which is equivalent to 25%.

Overview of the Standards Chapters (K-12)

Table I: Progression to Algebra I and Mathematics I in Kindergarten through Grade Eight

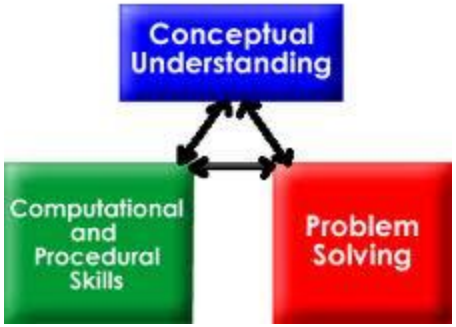
Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five	Grade Six	Grade Seven	Grade Eight
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent and solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply and divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities*

Group Discussion

Shift #2: Coherence: Think across grades, link to major topics within grades

In your groups, discuss what coherence in the math curriculum means to you. Be sure to address both elements—coherence within the grade and coherence across grades. Cite specific examples.

Rigor



The CCSSM require a balance of:

- Solid conceptual understanding
- Procedural skill and fluency
- Application of skills in problem solving situations

Pursuit of all three requires equal intensity in time, activities, and resources.

Content of the *Mathematics Framework,* K–5

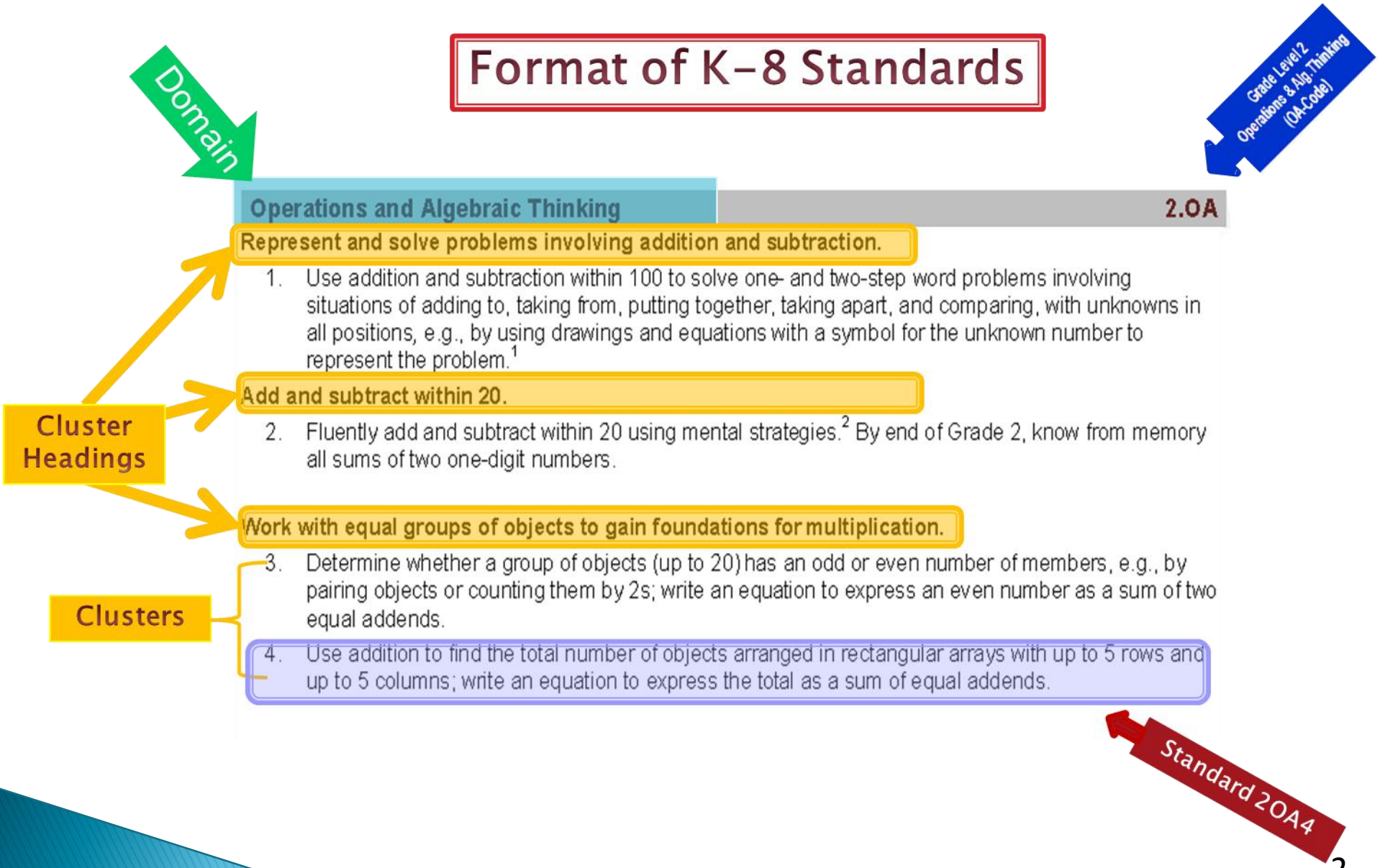
Focus Coherence Rigor

- A focus on understanding addition, subtraction, multiplication, and division (the four operations)
- Building from whole numbers in K–2 to fractions in grades 3–5
- Expectations of fluency with whole numbers and fractions

Required Fluencies in K–6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

Format of K–8 Standards



Grade Level Introduction

Grade Level Focus

WHAT STUDENTS LEARN IN GRADE TWO

In grade two, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Students also work towards fluency with addition and subtraction within 100. By the end of grade two, students know from memory all sums of two one-digit numbers.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting by fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction; and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Description

Grade 2– Cluster –Level Emphasis

Operations and Algebraic Thinking
[m]: Represent and solve problems involving addition and subtraction. **(2.OA.1▲)**
[m]: Add and subtract within 20. **(2.OA.2▲)**
[a/s]: Work with equal groups of objects to gain foundations for multiplication. **(2.OA.3–4)**

Number and Operations in Base Ten
[m]: Understand place value. **(2.NBT.1–4▲)**
[m]: Use place value understanding and properties of operations to add and subtract. **(2.NBT.5–9▲)**

Measurement and Data
[m]: Measure and estimate lengths in standard units. (2.MD.1–4▲)
[m]: Relate addition and subtraction to length. (2.MD.5–6▲)
[a/s]: Work with time and money. (2.MD.7–8)
[a/s]: Represent and interpret data. (2.MD.9–10)

Geometry
[a/s]: Reason with shapes and their attributes. (2.G.1–3)

Explanations of Major, Additional and Supporting Cluster–Level Emphases

Major [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.

Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis.

Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

Connecting Mathematical Practices and Content

MP Standards	Explanation and Examples
MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They ma use concrete objects or pictures to help them conceptualize and solve problem They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach. An example for this might be giving a student an equation and havir him/her write a story to match.
MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length. In second grade students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria? " Students translate the situation into an equation, such as: 25 + 17 = __ and then solve the problem. Students also contextualize situations during the problem solving process. For example, while solving the task above students might refer to the context of the task to determine that they need to subtract 19 if 19 children leave.
MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For exampl to solve 74 - 18, students may use a variety of strategies, and after working on the task, they might discuss and critique each others' reasoning and strategies citing similarities and differences between various problem-solving approaches
MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including writing numbers, using words (mathematical language) drawing pictures, using objects, acting out, making a chart or list, creatir equations. Students need opportunities to connect the different representations and explain the connections. They should be able to us any of these representations as needed. In grade two students model real-life mathematical situations with a number sentence or an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulatives and pictorial representations to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for th equation 43 + 17 = ___ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"

MP Standards	Explanation and Examples
MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited than others. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation. Students may use tools such as snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.
MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning Second grade students communicate clearly, using grade-level appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, while measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
MP.7. Look for and make use of structure.	Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles). Second grade students look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect skip counting by 10s to corresponding numbers on a 100s chart. Students see structure in the base-ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as a missing addend problems (e.g., 50 - 33 = __ can be written as 33 + __ = 50 and can be thought of as "How much more do I need to add to 33 to get to 50?")
MP.8. Look for and express regularity in repeated reasoning.	Second grade students notice repetitive actions in counting and computation (e.g., number patterns to skip count) When children have multiple opportunities to add and subtract, they look for shortcuts, such as using estimation strategies and then adjust the answer to compensate. Students continually check for the reasonableness of their solutions during and after completing a task by asking themselves, "Does this make sense?"

Grade Level 2 Overview

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

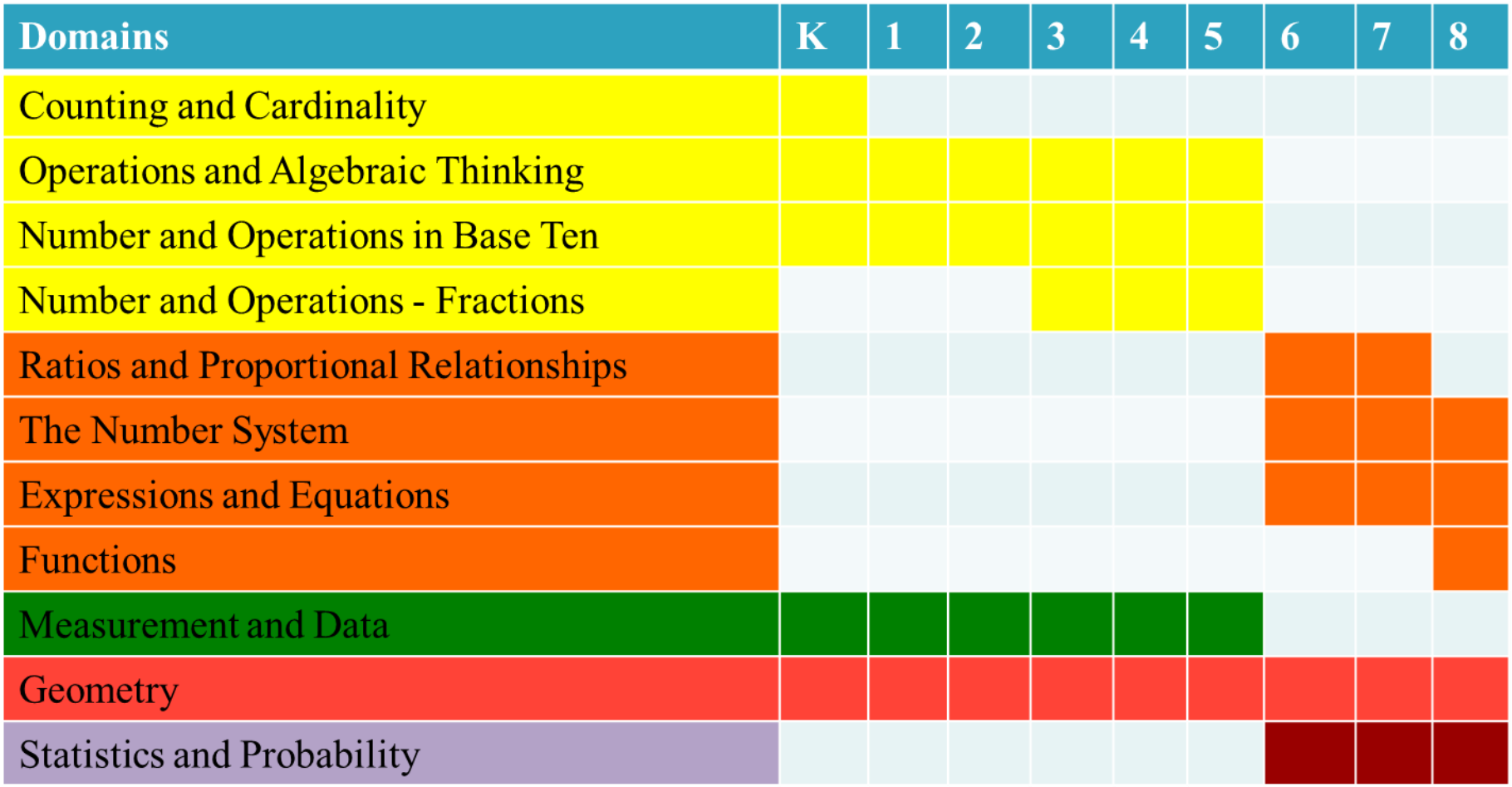
Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

K-8 Mathematics Domain Progressions



When you examine this grade level progression, what does it tell you?

Some K–5 examples:

WHAT STUDENTS LEARN IN GRADE ONE

Grade One Critical Areas of Instruction

- In grade one instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of and composing and decomposing geometric shapes. (excerpt from Grade 1)

Some K–5 examples:

Focus, Coherence, and Rigor:

- As students use various counting strategies when they participate in counting activities they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (**MP.4**), the use of precise language (**MP.6**), and repeated reasoning to find a solution (**MP.8**). (excerpt from Kindergarten)

Some K–5 examples:

Focus, Coherence, and Rigor

- Students' work with concepts of angle measures (**4.MD.5a and 7**) also connects to and supports adding fractions, which is major work at the grade in the cluster "Building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**4.NF.3–4▲**). For example, a one degree measure is a fraction of an entire rotation and adding angle measures together is the same as adding fractions with a denominator of 360. (excerpt from Grade 4)

Some K–5 examples:

Common Misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that $0.2 < 0.007$ simply because $2 < 7$.
- Students sometimes think the longer the decimal number the greater the value. For example, they think that 0.03 is greater than 0.3. (excerpt from Grade 4)

Content of the *Mathematics Framework,* 6–8

Focus Coherence Rigor

- A focus on ratio, rates, percent, and statistics and probability
- Extending operations with fractions to rational numbers
- Expectations of fluency with expressions and linear equations

Some 6–8 examples:

WHAT STUDENTS LEARN IN GRADE EIGHT

Grade Eight Critical Areas of Instruction

In grade eight, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem. (excerpt from Grade 8)

Some 6–8 examples:

Focus, Coherence, and Rigor

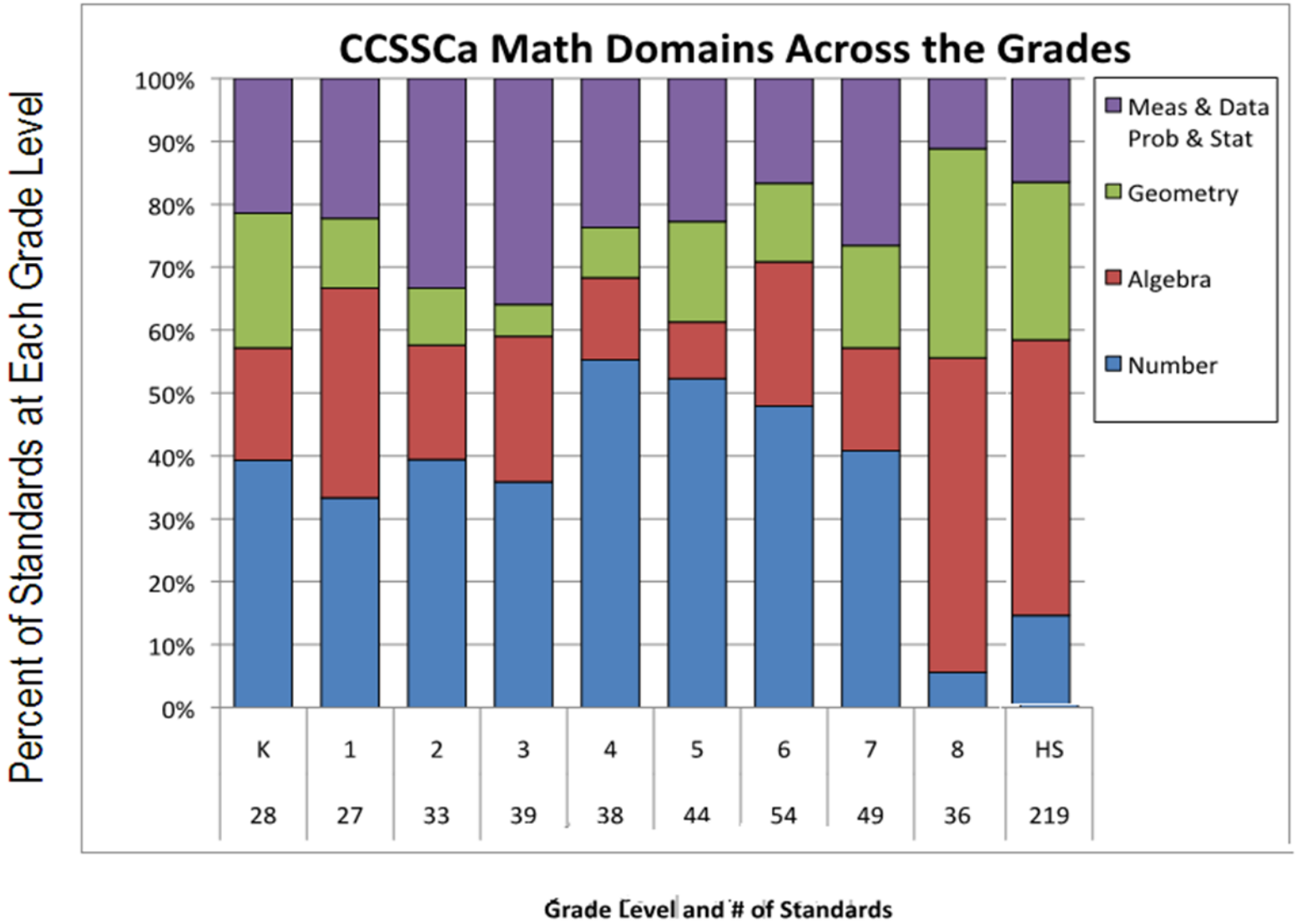
Proportional relationship problems support mathematical practices as students make sense of problems (**MP.1**), reason abstractly and quantitatively (**MP.2**), and model proportional relationships (**MP.4**). For example, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building for modeling purposes. (excerpt from Grade 7)

Some 6–8 examples:

Common Misconceptions:

Students may confuse dividing a quantity **by** ½ with dividing a quantity **in half**. Dividing **by** ½ is finding how many ½ sized portions there are, as in "dividing 7 **by** ½" which is $7 \div ½ = 14$. On the other hand, to divide a quantity **in half** is to divide the quantity into two parts equally, as in "dividing 7 **in half**" yields $7/2 = 3.5$. Students should understand that dividing **in half** is the same as dividing by two. (excerpt from Grade 6)

Standards Across the Grades



Taken from Michelle Lenertz's presentation, Coordinator, SBCSS

California Common Core State Standards

2 types of standards

Mathematical Content Standards

K-8th Content Standards

- Kinder
- 1st Grade
- 2nd Grade
- 3rd Grade
- 4th Grade
- 5th Grade
- 6th Grade
- 7th Grade
- 8th Grade
- Glossary

High School Content Standards

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics & Probability
- Glossary

Appendix A

Mathematical Practice Standards (K–12)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Different at each grade level

Recurring practices Grades K-12

High School Organization: Conceptual Categories, grades 9–12

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Modeling (*)
- Statistics and Probability (S)

High School Standards

- **Conceptual Categories**
  - Across course boundaries
  - Span high school years

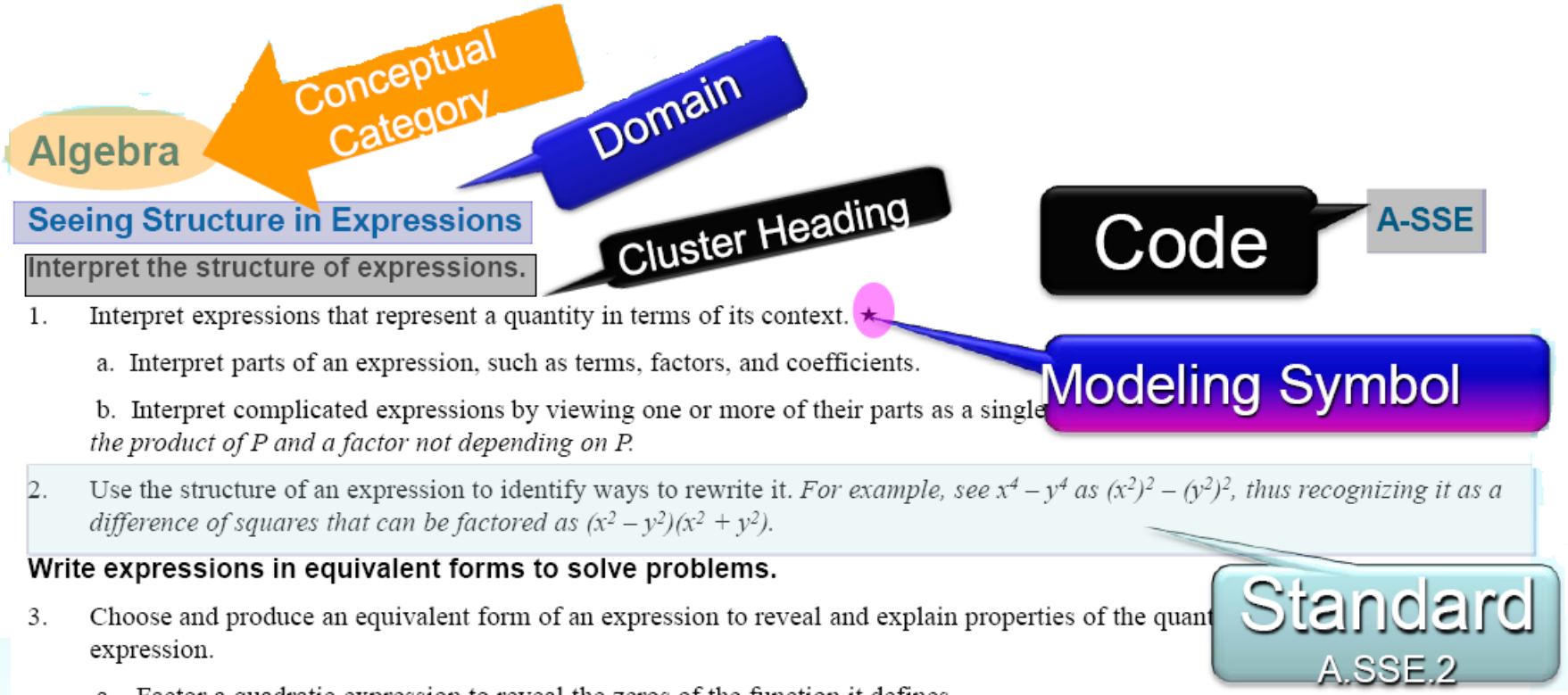
- **Standards**
  - "Core" for common mathematics curriculum for ALL students to be college and career ready
  - "College Ready" for entry level credit courses

(+) Additional courses that students should learn in order to take courses such as pre-calculus, calculus, Advanced Placement Calculus and Advanced Placement Statistics

Content of the *Mathematics Framework,* Higher Mathematics

- Traditional Pathway (Algebra I, Geometry, Algebra II)
- Integrated Pathway (Mathematics I, II, and III)
- Precalculus
- Statistics and Probability
- Calculus
- AP Probability and Statistics
- Mathematical Modeling

Format of High School Standards



Numbers and Quantity

- Extend the Real Numbers to include work with rational exponents and study of the properties of rational and irrational numbers
- Use quantities and quantitative reasoning to solve problems.

Numbers and Quantity

Required for higher math and/or + standards

- Compute with and use the Complex Numbers, use the Complex Number plane to represent numbers and operations
- Represent and use vectors
- Compute with matrices
- Use vector and matrices in modeling

Algebra and Functions

- Two separate conceptual categories
- Algebra category contains most of the typical "symbol manipulation" standards
- Functions category is more conceptual
- The two categories are interrelated

4

Algebra

- Creating, reading, and manipulating expressions
  - Understanding the structure of expressions
  - Includes operating with polynomials and simplifying rational expressions
- Solving equations and inequalities
  - Symbolically and graphically

Algebra

Required for higher math and/or + standards

- Expand a binomial using the Binomial Theorem
- Represent a system of linear equations as a matrix equation
- Find the inverse if it exists and use it to solve a system of equations

Functions

- Understanding, interpreting, and building functions
  - Includes multiple representations
- Emphasis is on linear and exponential models
- Extends trigonometric functions to functions defined in the unit circle and modeling periodic phenomena

Functions

Required for higher math and/or + standards

- Graph rational functions and identify zeros and asymptotes
- Compose functions
- Prove the addition and subtraction formulas for trigonometric functions and use them to solve problems

Functions

Required for higher math and/or + standards

- Inverse functions
  - Verify functions are inverses by composition
  - Find inverse values from a graph or table
  - Create an invertible function by restricting the domain
  - Use the inverse relationship between exponents and logarithms and in trigonometric functions

Modeling

Modeling has no specific domains, clusters or standards. Modeling is included in the other conceptual categories and marked with a asterisk.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Technology is valuable in modeling.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object.

Modeling

- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.

Geometry

- Understanding congruence
- Using similarity, right triangles, and trigonometry to solve problems

Congruence, similarity, and symmetry are approached through geometric transformations

Geometry

- Circles
- Expressing geometric properties with equations
  - Includes proving theorems and describing conic sections algebraically
- Geometric measurement and dimension
- Modeling with geometry

Geometry

Required for higher math and/or + standards

- Non-right triangle trigonometry
- Derive equations of hyperbolas and ellipses given foci and directrices
- Give an informal argument using Cavalieri's Principle for the formulas for the volume of solid figures

Statistics and Probability

- Analyze single a two variable data
- Understand the role of randomization in experiments
- Make decisions, use inference and justify conclusions from statistical studies
- Use the rules of probability

Interrelationships

- Algebra and Functions
  - Expressions can define functions
  - Determining the output of a function can involve evaluating an expression
- Algebra and Geometry
  - Algebraically describing geometric shapes
  - Proving geometric theorems algebraically

Content of the *Mathematics Framework,* Higher Mathematics

What Students Learn in Mathematics I

In Mathematics I, students continue their work with expressions and modeling and analyzing situations. In earlier grades, students informally define, evaluate, and compare functions, and use them to model relationships between quantities. In Mathematics I, students will learn function notation and develop the concepts of domain and range. (excerpt from Mathematics I)

Content of the *Mathematics Framework*, Higher Mathematics

Examples of Key Advances from Previous Grades or Courses

Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (6.EE.5). This perspective continues throughout Algebra I and Algebra II (A-REI).4 "Reasoned solving" plays a role in Algebra II because the equations students encounter can have extraneous solutions (A-REI.2). (excerpt from Algebra II)

Content of the *Mathematics Framework,* Higher Mathematics

Focus Coherence Rigor

- A focus on the mathematics that students need for success in college and careers
- Extending from algebraic concepts to calculus, trigonometry, and advanced statistics
- Expectation that students are college and career ready and able to utilize mathematics in their lives

Content of the *Mathematics Framework,* Higher Mathematics

MP.6. Attend to Precision.
Students begin to understand that a *rational number* has a specific definition, and that *irrational numbers* exist. They make use of the definition of *function* when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"
(excerpt from Mathematics II)

Content of the *Mathematics Framework,* Higher Mathematics

Throughout the Mathematics II chapter, the examples given will be framed as much as possible as modeling situations, to serve as illustrations of the concept of mathematical modeling. The big ideas of quadratic functions, graphing, solving equations, and rates of change will be explored through this lens. (excerpt from Mathematics II)

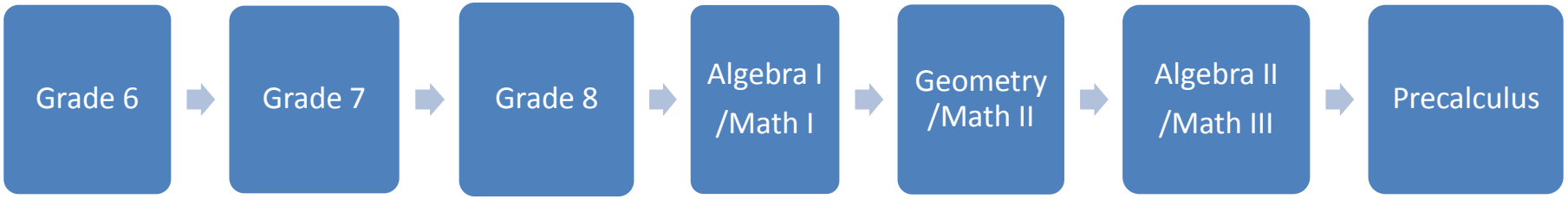
Course Sequences for Higher Mathematics

SBE Guidelines state: include a "discussion of options for middle school acceleration to support Algebra I or Integrated Mathematics I prior to ninth grade that are consistent with other Common Core states."

Acceleration decision points at middle school—between sixth and seventh grade—and in high school, after grade eight

- Acceleration in middle school
- Doubling up, enhanced pathway, or summer bridge in high school

Course Sequences for Higher Mathematics: No Acceleration



Possible Course Progressions from the Standards Document

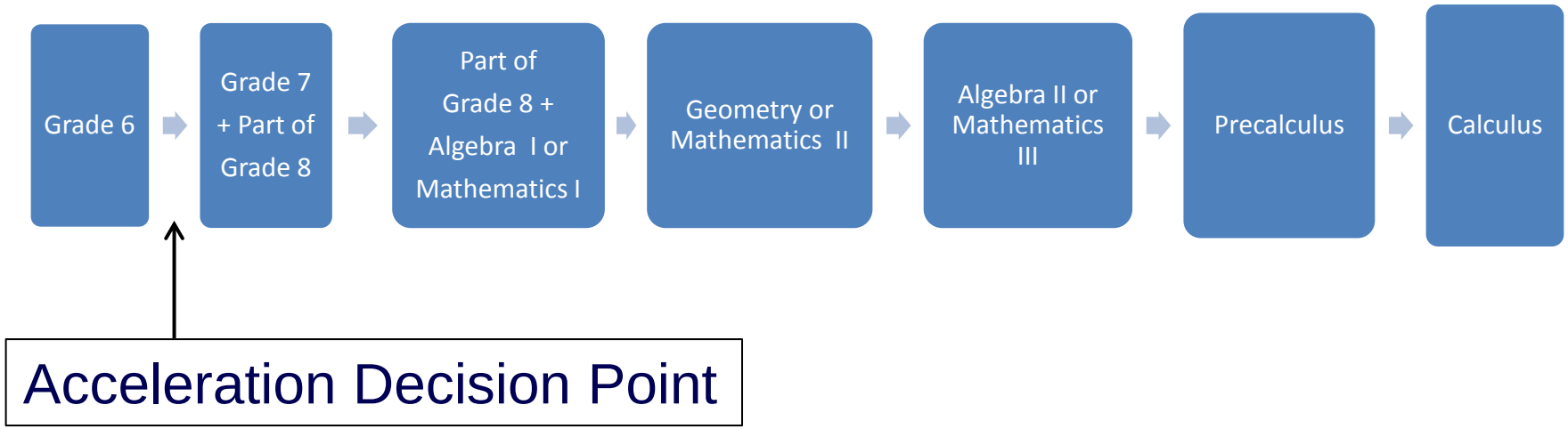
statistics, or discrete mathematics, is indicated by a plus symbol (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

Table 1: Model Mathematics Courses, by Grade Level

Discipline	Grade Seven	Grade Eight	Grade Nine	Grade Ten	Grade Eleven	Grade Twelve
Algebra I/Mathematics I						
Geometry/Mathematics II						
Algebra II/Mathematics III						
Advanced Placement Probability and Statistics						
Calculus						

Local districts determine which course offerings and sequences best meet the needs of students. The table above provides guidance on possible course-taking sequences in higher mathematics. It is not intended to be an exhaustive list of courses or

Course Sequences for Higher Mathematics: Middle School Acceleration



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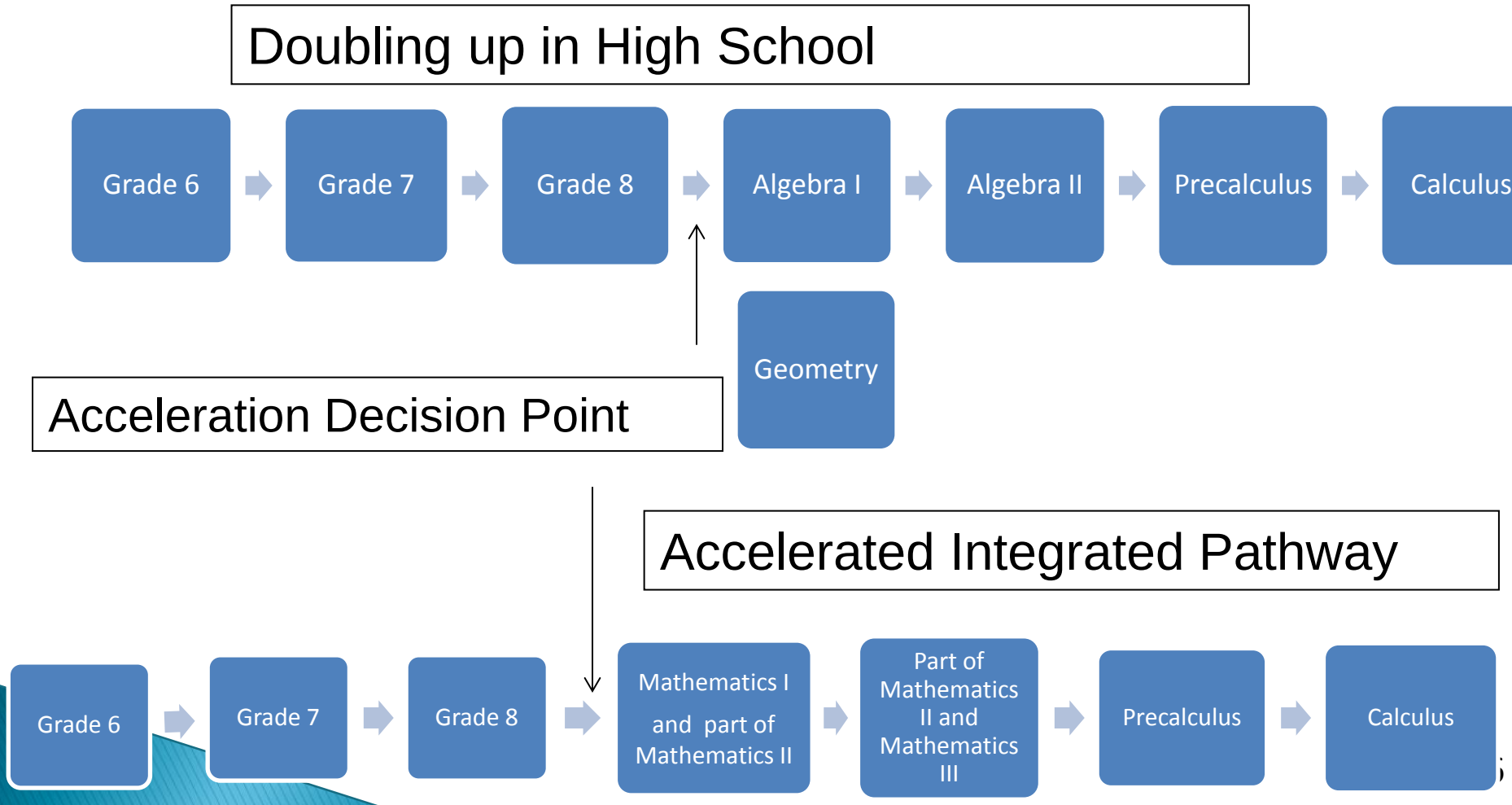
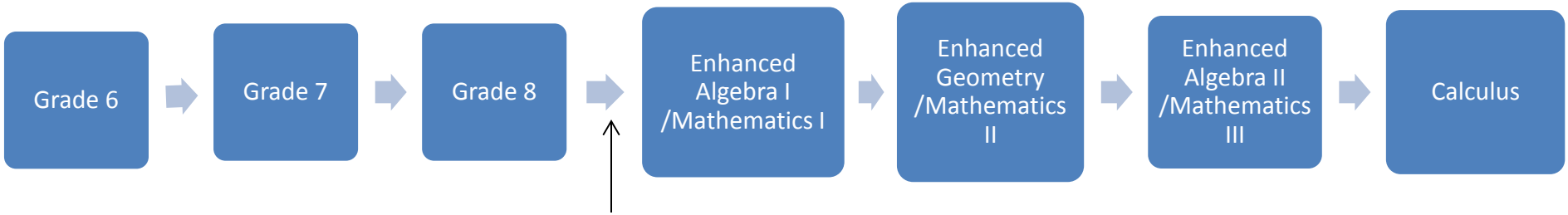
Course Sequences for Higher Mathematics: Doubling Up



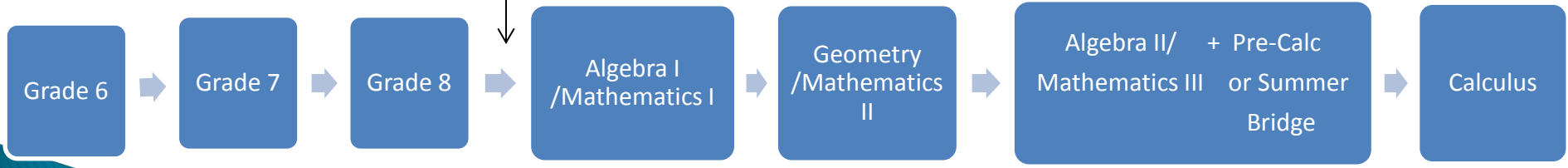
Course Sequences for Higher Mathematics: Enhanced & Summer Bridge

Enhanced Pathway



Acceleration Decision Point

Summer Bridge Pathway



Algebra I Graduation Requirement

Education Code 51224.5.

(b) Commencing with the 2003–04 school year and each year thereafter, at least one course, or a combination of the two courses, in mathematics required to be completed pursuant to subparagraph (B) of paragraph (1) of subdivision (a) of Section 51225.3 by pupils while in grades 9 to 12, inclusive, prior to receiving a diploma of graduation from high school, shall meet or exceed the rigor of the content standards for Algebra I, as adopted by the State Board of Education pursuant to Section 60605.

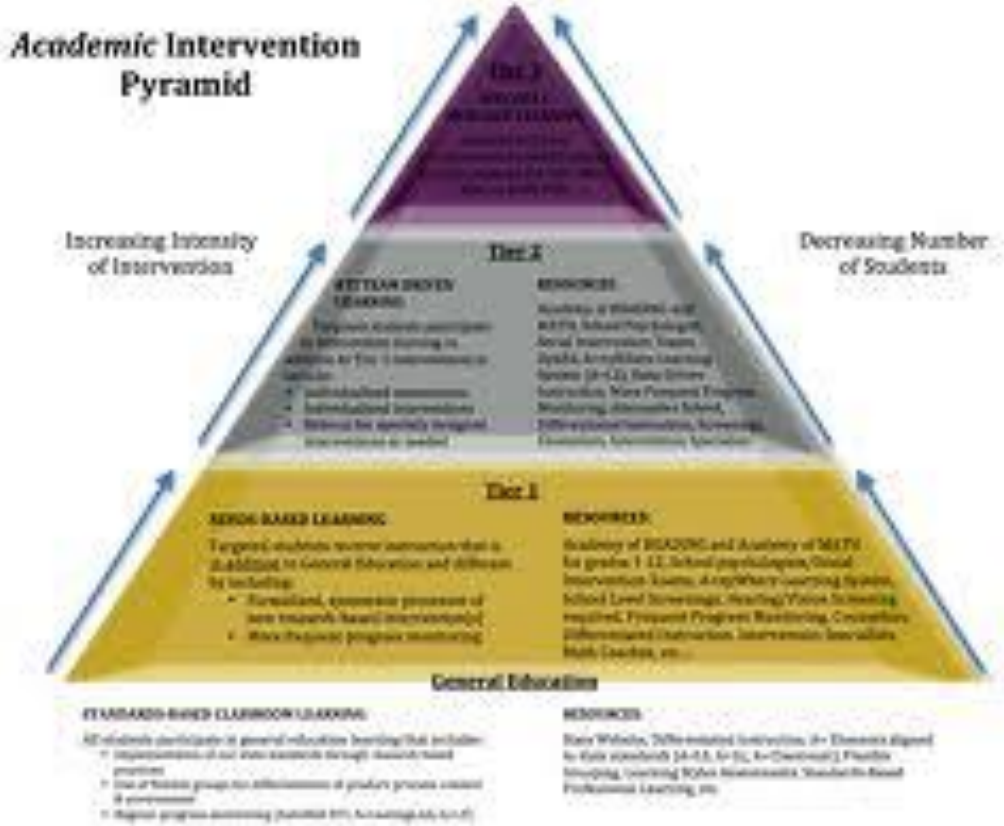
2014 Mathematics Instructional Materials Adoption

- Evaluation Criteria
  - Approved by the SBE January 2013
- Programs
  - K–8 and Algebra I/Mathematics I
  - 35 program submissions
  - 30 recommended for adoption by the Review Panels
- Over 100 IMRs and CREs participated

Universal Access

Source: http://www.teachertube.com/viewVideo.php?video_id=221481

Multi-Tiered Systems of Support



http://www.youtube.com/watch?v=aK-rBRvOXt8

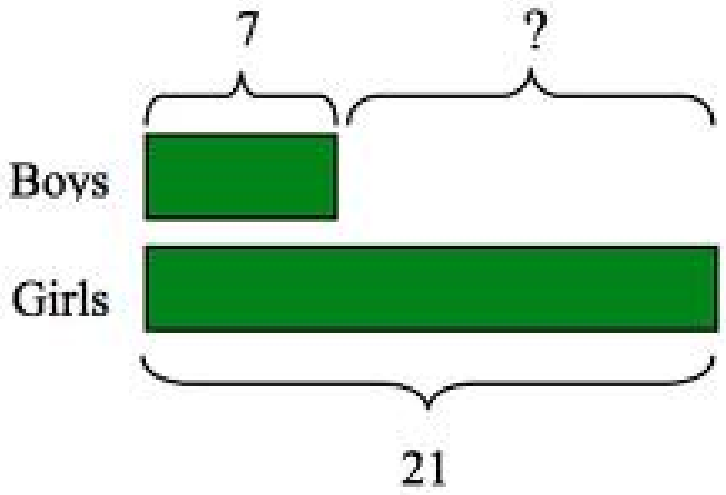
Instructional Models For Math

5 E Model
(Interactive)

http://www.youtube.com/watch?v=DfEcGkABK2A

Instructional Models For Math

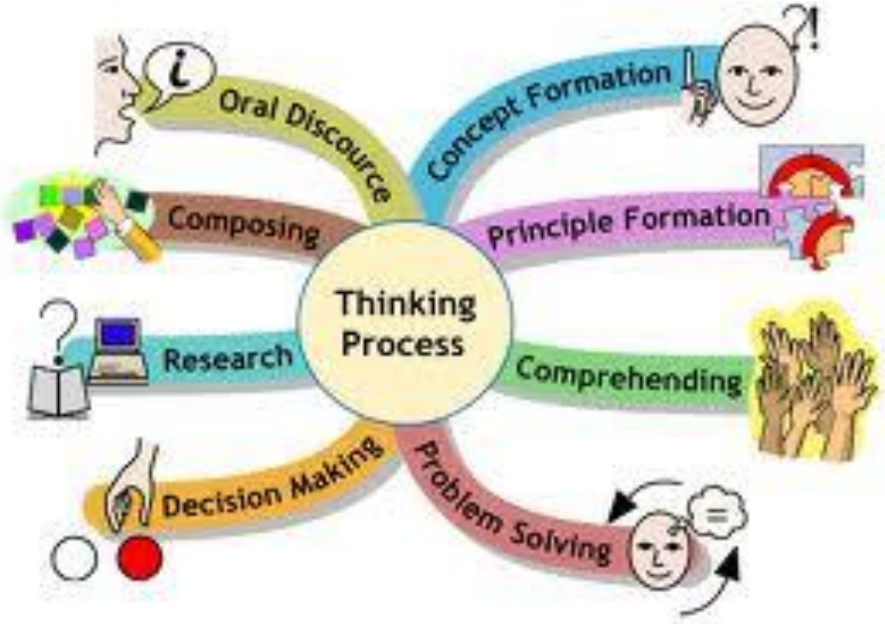
Singapore Math Model



http://www.youtube.com/watch?v=9Mrz1t2PvDQ

Instructional Models For Math

Concept Attainment Model



http://www.youtube.com/watch?v=l1ryvobIqIY

Instructional Models For Math

Cooperative Learning Model

http://www.youtube.com/watch?v=rWEwv_qobpU

0:13 to 3:45

Instructional Models For Math

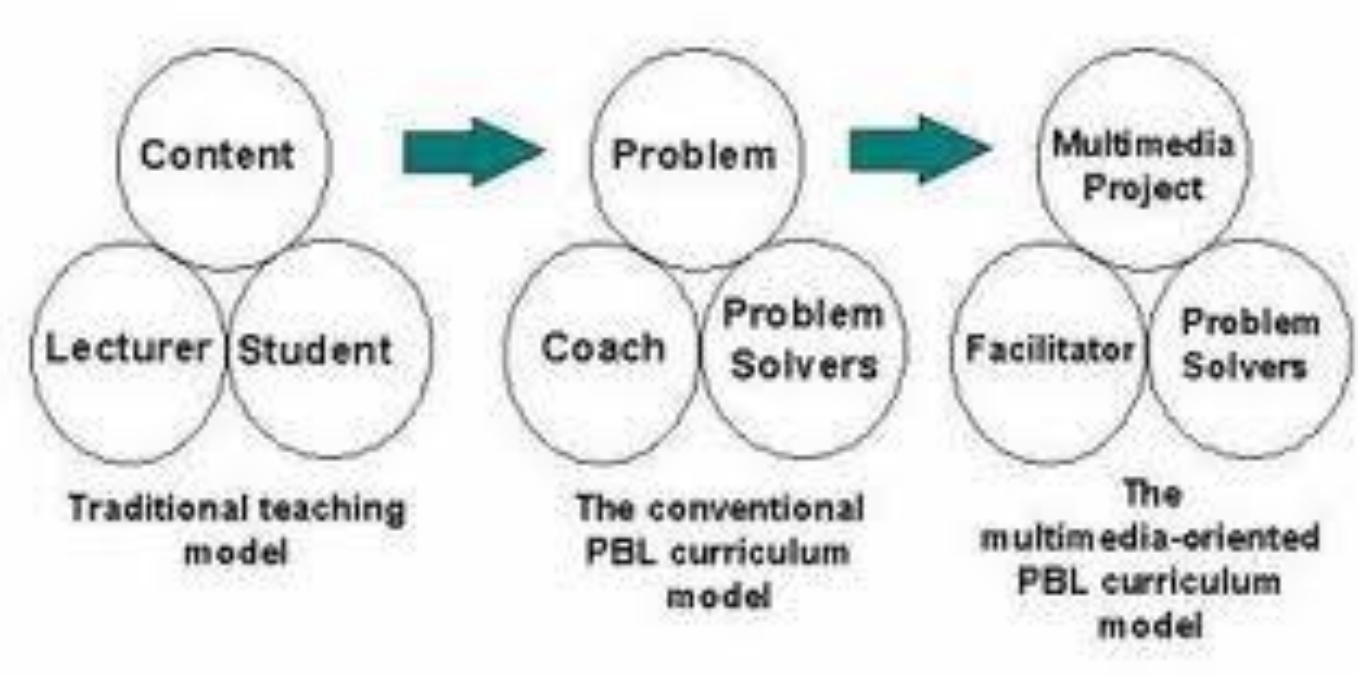
Cognitive Guided Instruction (CGI)

https://www.youtube.com/watch?v=ReBM2ffOv8I&list=PL15830F9AB2A059AA

0:59 to 4:06

Instructional Models For Math

Problem Based Learning



http://www.youtube.com/watch?v=lZS2MbxBGCM

Time: 0 to 2:30

2014 Mathematics Instructional Materials Adoption

Steps:

- **November 21–22, 2013:** Instructional Quality Commission acts on adoption recommendations from the Review Panels
- **January 15 –16, 2014:** State Board acts on adoption recommendations from the Instructional Quality Commission

Questions?